[redacted]

Summer Assignment for Precalculus

1. Read the following pages provided in this packet:

   p.3-10, 16-22, 42-47

2. On pages 82-85 in this packet do the following problems. Show all work on notebook paper. Be sure to number the problems and circle your answers.

   (1, 4, 8, 13, 15, 17, 21, 22, 25-29, 31, 33, 35-38 a, c only, 39, 41, 44, 69-72, 74-84 evens)

1.1 Lines in the Plane

The Slope of a Line

In this section, you will study lines and their equations. The slope of a nonvertical line represents the number of units the line rises or falls vertically for each unit of horizontal change from left to right. For instance, consider the two points (x_1, y_1) and (x_2, y_2) on the line shown in Figure 1.1. As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. That is,

$y_2 - y_1 =$ the change in y

and

$x_2 - x_1 =$ the change in x.

The slope of the line is given by the ratio of these two changes.

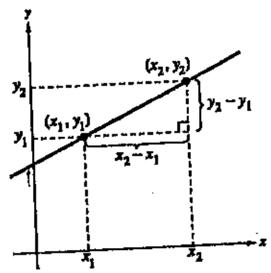


Figure 1.1

Definition of the Slope of a Line

The slope m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

When this formula for slope is used, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2). However, once you have done this, you must form the numerator and denominator using the same order of subtraction.

$$\underbrace{m = \frac{y_2 - y_1}{x_2 - x_1}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_1 - y_2}{x_1 - x_2}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_2 - y_1}{x_1 - x_2}}_{\text{Incorrect}}$$

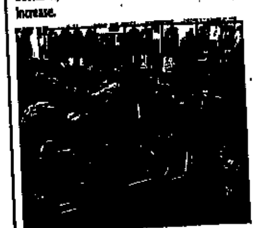
Throughout this text, the term *line* always means a *straight* line.

What you should learn

- Find the slopes of lines.
- Write linear equations given points on lines and their slopes.
- Use slope-intercept forms of linear equations to sketch lines.
- Use slope to identify parallel and perpendicular lines.

Why you should learn it

The slope of a line can be used to solve real-life problems. For instance, Exercise 68 on page 13 shows how to use slope to determine the years in which the earnings per share of stock for Harley-Davidson, Inc. showed the greatest and smallest increase.



Dwayne Newton/PhotoEdit

Example 1 Finding the Slope of a Line

Find the slope of the line passing through each pair of points.

a. $(-2, 0)$ and $(3, 1)$ b. $(-1, 2)$ and $(2, 2)$ c. $(0, 4)$ and $(1, -1)$

Solution

a. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - (-2)} = \frac{1}{3 + 2} = \frac{1}{5}$ (Difference in y-values / Difference in x-values)

b. $m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0$

c. $m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5$

The graphs of the three lines are shown in Figure 1.2. Note that the square setting gives the correct "steepness" of the lines.

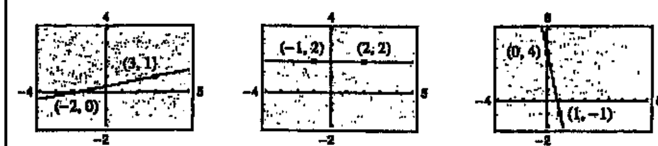


Figure 1.2

✓ *Checkpoint* Now try Exercise 9.

The definition of slope does not apply to vertical lines. For instance, consider the points $(3, 4)$ and $(3, 1)$ on the vertical line shown in Figure 1.3. Applying the formula for slope, you obtain

$$m = \frac{4 - 1}{3 - 3} = \frac{3}{0}. \qquad \text{Undefined}$$

Because division by zero is undefined, the slope of a vertical line is undefined.

From the slopes of the lines shown in Figures 1.2 and 1.3, you can make the following generalizations about the slope of a line.

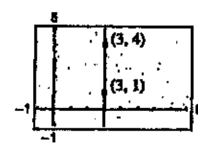


Figure 1.3

The Slope of a Line

1. A line with positive slope $(m > 0)$ *rises* from left to right.
2. A line with negative slope $(m < 0)$ *falls* from left to right.
3. A line with zero slope $(m = 0)$ is *horizontal*.
4. A line with undefined slope is *vertical*.

Exploration

Use a graphing utility to compare the slopes of the lines $y = 0.5x$, $y = x$, $y = 2x$, and $y = 4x$. What do you observe about these lines? Compare the slopes of the lines $y = -0.5x$, $y = -x$, $y = -2x$, and $y = -4x$. What do you observe about these lines? (*Hint:* Use a square setting to guarantee a true geometric perspective.)

he Point-Slope Form of the Equation of a Line

: you know the slope of a line *and* you also know the coordinates of one point n the line, you can find an equation for the line. For instance, in Figure 1.4, let $_1, y_1)$ be a point on the line whose slope is m. If (x, y) is any *other* point on the ne, it follows that

$$\frac{y - y_1}{x - x_1} = m.$$

his equation in the variables x and y can be rewritten in the point-slope form of ıe equation of a line.

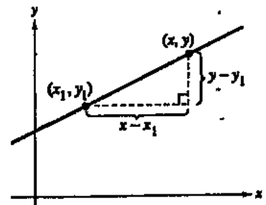


Figure 1.4

Point-Slope Form of the Equation of a Line

The point-slope form of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

The point-slope form is most useful for finding the equation of a line if you now at least one point that the line passes through and the slope of the line. You hould remember this form of the equation of a line.

:xample 2 The Point-Slope Form of the Equation of a Line

'ind an equation of the line that passes through the point $(1, -2)$ and has a slope f 3.

ıolution

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = 3(x - 1)$	Substitute for y_1, m, and x_1.
$y + 2 = 3x - 3$	Simplify.
$y = 3x - 5$	Solve for y.

The line is shown in Figure 1.5.

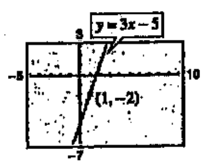


Figure 1.5

✓ *Checkpoint* Now try Exercise 25.

STUDY TIP

When you find an equation of the line that passes through two given points, you need to substitute the coordinates of only one of the points into the point-slope form. It does not matter which point you choose because both points will yield the same result.

The point-slope form can be used to find an equation of a nonvertical line ›assing through two points (x_1, y_1) and (x_2, y_2). First, find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

This is sometimes called the two-point form of the equation of a line.

Sketching Graphs of Lines

Many problems in coordinate geometry can be classified as follows.

1. Given a graph (or parts of it), find its equation.
2. Given an equation, sketch its graph.

For lines, the first problem is solved easily by using the point-slope form. This formula, however, is not particularly useful for solving the second type of problem. The form that is better suited to graphing linear equations is the slope-intercept form of the equation of a line, $y = mx + b$.

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

Example 4 Using the Slope-Intercept Form

Determine the slope and y-intercept of each linear equation. Then describe its graph.

a. $x + y = 2$ b. $y = 2$

Algebraic Solution

a. Begin by writing the equation in slope-intercept form.

$x + y = 2$	Write original equation.
$y = 2 - x$	Subtract x from each side.
$y = -x + 2$	Write in slope-intercept form.

From the slope-intercept form of the equation, the slope is -1 and the y-intercept is $(0, 2)$. Because the slope is negative, you know that the graph of the equation is a line that falls one unit for every unit it moves to the right.

b. By writing the equation $y = 2$ in slope-intercept form

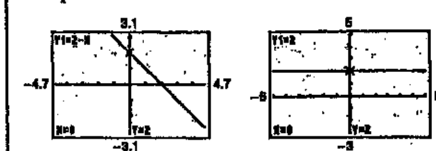
$$y = (0)x + 2$$

you can see that the slope is 0 and the y-intercept is $(0, 2)$. A zero slope implies that the line is horizontal.

Graphical Solution

a. Solve the equation for y to obtain $y = 2 - x$. Enter this equation in your graphing utility. Use a decimal viewing window to graph the equation. To find the y-intercept, use the *value* or *trace* feature. When $x = 0$, $y = 2$, as shown in Figure 1.7(a). So, the y-intercept is $(0, 2)$. To find the slope, continue to use the *trace* feature. Move the cursor along the line until $x = 1$. At this point, $y = 1$. So the graph falls 1 unit for every unit it moves to the right, and the slope is -1.

b. Enter the equation $y = 2$ in your graphing utility and graph the equation. Use the *trace* feature to verify the y-intercept $(0, 2)$ as shown in Figure 1.7(b), and to see that the value of y is the same for all values of x. So, the slope of the horizontal line is 0.



(a) (b)
Figure 1.7

✓ *Checkpoint* Now try Exercise 45.

From the slope-intercept form of the equation of a line, you can see that a horizontal line ($m = 0$) has an equation of the form $y = b$. This is consistent with the fact that each point on a horizontal line through $(0, b)$ has a y-coordinate of b. Similarly, each point on a vertical line through $(a, 0)$ has an x-coordinate of a. So, a vertical line has an equation of the form $x = a$. This equation cannot be written in slope-intercept form because the slope of a vertical line is undefined. However, *every* line has an equation that can be written in the general form

$$Ax + By + C = 0 \qquad \text{General form of the equation of a line}$$

where A and B are not *both* zero.

Exploration

Graph the lines $y_1 = 2x + 1$, $y_2 = \frac{1}{2}x + 1$, and $y_3 = -2x + 1$ in the same viewing window. What do you observe?

Graph the lines $y_1 = 2x + 1$, $y_2 = 2x$, and $y_3 = 2x - 1$ in the same viewing window. What do you observe?

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

Example 5 Different Viewing Windows

The graphs of the two lines

$$y = -x - 1 \quad \text{and} \quad y = -10x - 1$$

are shown in Figure 1.8. Even though the slopes of these lines are quite different (-1 and -10, respectively), the graphs seem misleadingly similar because the viewing windows are different.

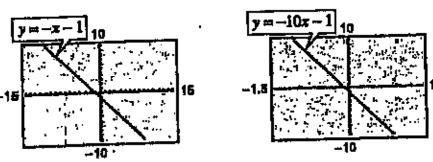


Figure 1.8

✓ *Checkpoint* Now try Exercise 49.

TECHNOLOGY TIP When a graphing utility is used to graph a line, it is important to realize that the graph of the line may not visually appear to have the slope indicated by its equation. This occurs because of the viewing window used for the graph. For instance, Figure 1.9 shows graphs of $y = 2x + 1$ produced on a graphing utility using three different viewing windows. Notice that the slopes in Figures 1.9(a) and (b) do not visually appear to be equal to 2. However, if you use a *square setting*, as in Figure 1.9(c), the slope visually appears to be 2.

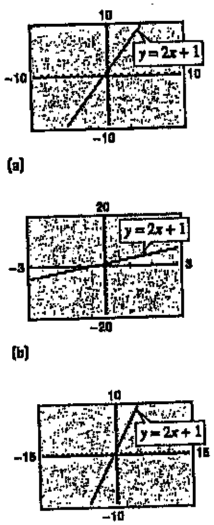


Figure 1.9

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular.

Parallel Lines

Two distinct nonvertical lines are parallel if and only if their slopes are equal. That is,

$$m_1 = m_2.$$

Example 6 Equations of Parallel Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is parallel to the line $2x - 3y = 5$.

Solution

Begin by writing the equation of the given line in slope-intercept form.

$$2x - 3y = 5 \qquad \text{Write original equation.}$$
$$-2x + 3y = -5 \qquad \text{Multiply by } -1.$$
$$3y = 2x - 5 \qquad \text{Add } 2x \text{ to each side.}$$
$$y = \frac{2}{3}x - \frac{5}{3} \qquad \text{Write in slope-intercept form.}$$

Therefore, the given line has a slope of $m = \frac{2}{3}$. Any line parallel to the given line must also have a slope of $\frac{2}{3}$. So, the line through $(2, -1)$ has the following equation.

$$y - (-1) = \frac{2}{3}(x - 2) \qquad \text{Write in point-slope form.}$$
$$y + 1 = \frac{2}{3}x - \frac{4}{3} \qquad \text{Simplify.}$$
$$y = \frac{2}{3}x - \frac{7}{3} \qquad \text{Write in slope-intercept form.}$$

Notice the similarity between the slope-intercept form of the original equation and the slope-intercept form of the parallel equation. The graphs of both equations are shown in Figure 1.10.

✓ *Checkpoint* Now try Exercise 55(a).

Perpendicular Lines

Two nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$

TECHNOLOGY TIP

Be careful when you graph equations such as $y = \frac{2}{3}x - \frac{7}{3}$ on your graphing utility. A common mistake is to type in the equation as

Y1 = 2/3X − 7/3,

which may not be interpreted by your graphing utility as the original equation. You should use one of the following formulas.

Y1 = 2X/3 − 7/3

Y1 = (2/3)X − 7/3

Do you see why?

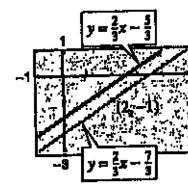


Figure 1.10

Example 7 Equations of Perpendicular Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is perpendicular to the line $2x - 3y = 5$.

Solution

From Example 6, you know that the equation can be written in the slope-intercept form $y = \frac{2}{3}x - \frac{5}{3}$. You can see that the line has a slope of $\frac{2}{3}$. So, any line perpendicular to this line must have a slope of $-\frac{3}{2}$ (because $-\frac{3}{2}$ is the negative reciprocal of $\frac{2}{3}$). So, the line through the point $(2, -1)$ has the following equation.

$$y - (-1) = -\tfrac{3}{2}(x - 2) \quad \text{Write in point-slope form.}$$

$$y + 1 = -\tfrac{3}{2}x + 3 \quad \text{Simplify.}$$

$$y = -\tfrac{3}{2}x + 2 \quad \text{Write in slope-intercept form.}$$

The graphs of both equations are shown in Figure 1.11.

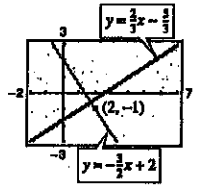


Figure 1.11

☑ *Checkpoint* Now try Exercise 55(b).

Example 8 Graphs of Perpendicular Lines

Use a graphing utility to graph the lines

$$y = x + 1$$

and

$$y = -x + 3$$

in the same viewing window. The lines are supposed to be perpendicular (they have slopes of $m_1 = 1$ and $m_2 = -1$). Do they appear to be perpendicular on the display?

Solution

If the viewing window is nonsquare, as in Figure 1.12, the two lines will not appear perpendicular. If, however, the viewing window is square, as in Figure 1.13, the lines will appear perpendicular.

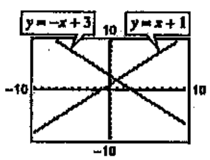


Figure 1.12

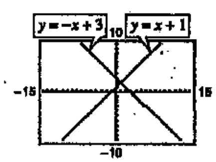


Figure 1.13

☑ *Checkpoint* Now try Exercise 61.

1.2 Functions

What you should learn

- Decide whether relations between two variables represent a function.
- Use function notation and evaluate functions.
- Find the domains of functions.
- Use functions to model and solve real-life problems.
- Evaluate difference quotients.

Why you should learn it

Many natural phenomena can be modeled by functions, such as the force of water against the face of a dam, explored in Exercise 81 on page 28.


Kunio Owaki/Corbis

Introduction to Functions

Many everyday phenomena involve pairs of quantities that are related to each other by some rule of correspondence. The mathematical term for such a rule of correspondence is a **relation**. Here are two examples.

1. The simple interest I earned on an investment of $1000 for 1 year is related to the annual interest rate r by the formula $I = 1000r$.
2. The area A of a circle is related to its radius r by the formula $A = \pi r^2$.

Not all relations have simple mathematical formulas. For instance, people commonly match up NFL starting quarterbacks with touchdown passes, and hours of the day with temperature. In each of these cases, there is some relation that matches each item from one set with exactly one item from a different set. Such a relation is called a **function**.

Definition of a Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

To help understand this definition, look at the function that relates the time of day to the temperature in Figure 1.14.

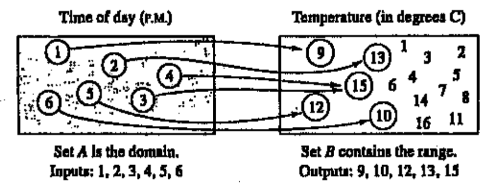


Figure 1.14

This function can be represented by the ordered pairs $\{(1, 9°), (2, 13°), (3, 15°), (4, 15°), (5, 12°), (6, 10°)\}$. In each ordered pair, the first coordinate (x-value) is the **input** and the second coordinate (y-value) is the **output**.

Characteristics of a Function from Set A to Set B

1. Each element of A must be matched with an element of B.
2. Some elements of B may not be matched with any element of A.
3. Two or more elements of A may be matched with the same element of B.
4. An element of A (the domain) cannot be matched with two different elements of B.

ibrary of Functions: Data Defined Function

lany functions do not have simple mathematical formulas, but are defined y real-life data. Such functions arise when you are using collections of data model real-life applications. Functions can be represented in four ways.

. *Verbally* by a sentence that describes how the input variable is related to the output variable

   Example: The input value x is the election year from 1952 to 2004 and the output value y is the elected president of the United States.

. *Numerically* by a table or a list of ordered pairs that matches input values with output values

   Example: In the set of ordered pairs $\{(2, 34), (4, 40), (6, 45), (8, 50), (10, 54)\}$, the input value is the age of a male child in years and the output value is the height of the child in inches.

3. *Graphically* by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis

   Example: See Figure 1.15.

4. *Algebraically* by an equation in two variables

   Example: The formula for temperature, $F = \frac{9}{5}C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius, is an equation that represents a function. You will see that it is often convenient to approximate data using a mathematical model or formula.

STUDY TIP

To determine whether or not a relation is a function, you must decide whether each input value is matched with exactly one output value. If any input value is matched with two or more output values, the relation is not a function.

:xample 1 Testing for Functions

)ecide whether the relation represents y as a function of x.

a.

Input x	2	2	3	4	5
Output y	11	10	8	5	1

b.

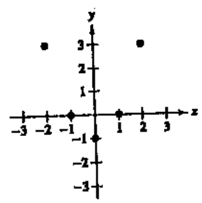


Figure 1.15

STUDY TIP

Be sure you see that the *range* of a function is not the same as the use of *range* relating to the viewing window of a graphing utility.

Solution

a. This table *does not* describe y as a function of x. The input value 2 is matched with two different y-values.

b. The graph in Figure 1.15 *does* describe y as a function of x. Each input value is matched with exactly one output value.

☑ ***Checkpoint*** Now try Exercise 5.

In algebra, it is common to represent functions by equations or formulas involving two variables. For instance, the equation $y = x^2$ represents the variable y as a function of the variable x. In this equation, x is the independent variable and y is the dependent variable. The domain of the function is the set of all values taken on by the independent variable x, and the range of the function is the set of all values taken on by the dependent variable y.

Exploration

Use a graphing utility to graph $x^2 + y = 1$. Then use the graph to write a convincing argument that each x-value has at most one y-value.

Use a graphing utility to graph $-x + y^2 = 1$. (*Hint:* You will need to use two equations.) Does the graph represent y as a function of x? Explain.

Example 2 Testing for Functions Represented Algebraically

Which of the equations represent(s) y as a function of x?

a. $x^2 + y = 1$ b. $-x + y^2 = 1$

Solution

To determine whether y is a function of x, try to solve for y in terms of x.

a. Solving for y yields

$x^2 + y = 1$ Write original equation.

$y = 1 - x^2$. Solve for y.

Each value of x corresponds to exactly one value of y. So, y is a function of x.

b. Solving for y yields

$-x + y^2 = 1$ Write original equation.

$y^2 = 1 + x$ Add x to each side.

$y = \pm\sqrt{1 + x}$. Solve for y.

The $\pm$ indicates that for a given value of x there correspond two values of y. For instance, when $x = 3$, $y = 2$ or $y = -2$. So, y is not a function of x.

☑ ***Checkpoint*** Now try Exercise 19.

Function Notation

When an equation is used to represent a function, it is convenient to name the function so that it can be referenced easily. For example, you know that the equation $y = 1 - x^2$ describes y as a function of x. Suppose you give this function the name "f." Then you can use the following function notation.

Input	*Output*	*Equation*
x	$f(x)$	$f(x) = 1 - x^2$

The symbol $f(x)$ is read as the *value of f at x* or simply *f of x*. The symbol $f(x)$ corresponds to the y-value for a given x. So, you can write $y = f(x)$. Keep in mind that f is the *name* of the function, whereas $f(x)$ is the *output value* of the function at the *input value* x. In function notation, the *input* is the independent variable and the *output* is the dependent variable. For instance, the function $f(x) = 3 - 2x$ has *function values* denoted by $f(-1), f(0)$, and so on. To find these values, substitute the specified input values into the given equation.

For $x = -1$, $f(-1) = 3 - 2(-1) = 3 + 2 = 5$.

For $x = 0$, $f(0) = 3 - 2(0) = 3 - 0 = 3$.

TECHNOLOGY TIP

You can use a graphing utility to evaluate a function. Use the Evaluating an Algebraic Expression Program found on the website *college.hmco.com*. The program will prompt you for a value of x, and then evaluate the expression in the equation editor for that value of x. Try using the program to evaluate several different functions of x.

Although f is often used as a convenient function name and x is often used as the independent variable, you can use other letters. For instance,

$$f(x) = x^2 - 4x + 7, \quad f(t) = t^2 - 4t + 7, \quad \text{and} \quad g(s) = s^2 - 4s + 7$$

all define the same function. In fact, the role of the independent variable is that of a "placeholder." Consequently, the function could be described by

$$f(\quad) = (\quad)^2 - 4(\quad) + 7.$$

Example 3 Evaluating a Function

Let $g(x) = -x^2 + 4x + 1$. Find (a) $g(2)$, (b) $g(t)$, and (c) $g(x + 2)$.

Solution

a. Replacing x with 2 in $g(x) = -x^2 + 4x + 1$ yields the following.

$$g(2) = -(2)^2 + 4(2) + 1 = -4 + 8 + 1 = 5$$

b. Replacing x with t yields the following.

$$g(t) = -(t)^2 + 4(t) + 1 = -t^2 + 4t + 1$$

c. Replacing x with $x + 2$ yields the following.

$$\begin{aligned} g(x+2) &= -(x+2)^2 + 4(x+2) + 1 && \text{Substitute } x + 2 \text{ for } x. \\ &= -(x^2 + 4x + 4) + 4x + 8 + 1 && \text{Multiply.} \\ &= -x^2 - 4x - 4 + 4x + 8 + 1 && \text{Distributive Property} \\ &= -x^2 + 5 && \text{Simplify.} \end{aligned}$$

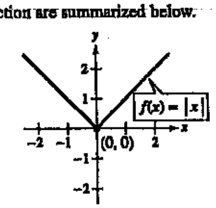
☑ *Checkpoint* Now try Exercise 33.

In Example 3, note that $g(x + 2)$ is not equal to $g(x) + g(2)$. In general, $g(u + v) \neq g(u) + g(v)$.

Library of Functions: Piecewise-Defined Function

A *piecewise-defined function* is a function that is defined by two or more equations over a specified domain. The *absolute value function* given by $f(x) = |x|$ can be written as a piecewise-defined function. The basic characteristics of the absolute value function are summarized below.

Graph of $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$

An illustration of a piecewise-defined function is given in Example 4.

Example 4 A Piecewise-Defined Function

Evaluate the function when $x = -1$ and 0.

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

Solution

Because $x = -1$ is less than 0, use $f(x) = x^2 + 1$ to obtain

$$f(-1) = (-1)^2 + 1 = 2.$$

For $x = 0$, use $f(x) = x - 1$ to obtain

$$f(0) = (0) - 1 = -1.$$

☑ *Checkpoint* Now try Exercise 37.

TECHNOLOGY TIP

Most graphing utilities can graph piecewise-defined functions. For instructions on how to enter a piecewise-defined function into your graphing utility, consult your user's manual. You may find it helpful to set your graphing utility to *dot mode* before graphing.

The Domain of a Function

The domain of a function can be described explicitly or it can be *implied* by the expression used to define the function. The implied domain is the set of all real numbers for which the expression is defined. For instance, the function

$$f(x) = \frac{1}{x^2 - 4} \qquad \text{Domain excludes } x\text{-values that result in division by zero.}$$

has an implied domain that consists of all real x other than $x = \pm 2$. These two values are excluded from the domain because division by zero is undefined. Another common type of implied domain is that used to avoid even roots of negative numbers. For example, the function

$$f(x) = \sqrt{x} \qquad \text{Domain excludes } x\text{-values that result in even roots of negative numbers.}$$

is defined only for $x \geq 0$. So, its implied domain is the interval $[0, \infty)$. In general, the domain of a function *excludes* values that would cause division by zero *or* result in the even root of a negative number.

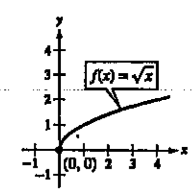
Exploration

Use a graphing utility to graph $y = \sqrt{4 - x^2}$. What is the domain of this function? Then graph $y = \sqrt{x^2 - 4}$. What is the domain of this function? Do the domains of these two functions overlap? If so, for what values?

Library of Functions: Radical Function

Radical functions arise from the use of rational exponents. The most common radical function is the *square root function* given by $f(x) = \sqrt{x}$. The basic characteristics of the square root function are summarized below.

Graph of $f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

STUDY TIP

Because the square root function is not defined for $x < 0$, you must be careful when analyzing the domains of complicated functions involving the square root symbol.

ample 5 Finding the Domain of a Function

d the domain of each function.

$f: \{(-3, 0), (-1, 4), (0, 2), (2, 2), (4, -1)\}$

$g(x) = -3x^2 + 4x + 5$ c. $h(x) = \dfrac{1}{x+5}$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$ e. $k(x) = \sqrt{4 - 3x}$

STUDY TIP

In Example 5(e), $4 - 3x \geq 0$ is a *linear inequality*. For help with solving linear inequalities, see Appendix E.

lution

The domain of f consists of all first coordinates in the set of ordered pairs.

Domain $= \{-3, -1, 0, 2, 4\}$

The domain of g is the set of all *real* numbers.

Excluding x-values that yield zero in the denominator, the domain of h is the set of all real numbers $x \neq -5$.

Because this function represents the volume of a sphere, the values of the radius r must be positive. So, the domain is the set of all real numbers r such that $r > 0$.

This function is defined only for x-values for which $4 - 3x \geq 0$. By solving this inequality, you will find that the domain of k is all real numbers that are less than or equal to $\frac{4}{3}$.

✓ *Checkpoint* Now try Exercise 51.

In Example 5(d), note that the *domain of a function may be implied by the hysical context.* For instance, from the equation $V = \frac{4}{3}\pi r^3$, you would have no ason to restrict r to positive values, but the physical context implies that a phere cannot have a negative or zero radius.

For some functions, it may be easier to find the domain and range of the inction by examining its graph.

xample 6 Finding the Domain and Range of a Function

Jse a graphing utility to find the domain and range of the function

$f(x) = \sqrt{9 - x^2}$.

olution

Graph the function as $y = \sqrt{9 - x^2}$, as shown in Figure 1.16. Using the *trace* eature of a graphing utility, you can determine that the x-values extend from -3 o 3 and the y-values extend from 0 to 3. So, the domain of the function f is all eal numbers such that $-3 \leq x \leq 3$ and the range of f is all real numbers such hat $0 \leq y \leq 3$.

✓ *Checkpoint* Now try Exercise 61.

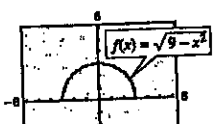


Figure 1.16

Applications

Example 7 Cellular Phone Subscribers

The number N (in millions) of cellular phone subscribers in the United States increased in a linear pattern from 1995 to 1997, as shown in Figure 1.17. Then, in 1998, the number of subscribers took a jump, and until 2001, increased in a *different* linear pattern. These two patterns can be approximated by the function

$$N(t) = \begin{cases} 10.75t - 20.1, & 5 \leq t \leq 7 \\ 20.11t - 92.8, & 8 \leq t \leq 11 \end{cases}$$

where t represents the year, with $t = 5$ corresponding to 1995. Use this function to approximate the number of cellular phone subscribers for each year from 1995 to 2001. (Source: Cellular Telecommunications & Internet Association)

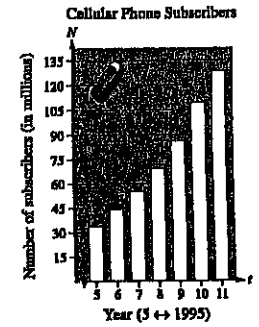


Figure 1.17

Solution

From 1995 to 1997, use $N(t) = 10.75t - 20.1$

$\underbrace{33.7}_{1995}, \underbrace{44.4}_{1996}, \underbrace{55.2}_{1997}$

From 1998 to 2001, use $N(t) = 20.11t - 92.8$.

$\underbrace{68.1}_{1998}, \underbrace{88.2}_{1999}, \underbrace{108.3}_{2000}, \underbrace{128.4}_{2001}$

✓ *Checkpoint* Now try Exercise 79.

Example 8 The Path of a Baseball

A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and an angle of 45°. The path of the baseball is given by the function

$f(x) = -0.0032x^2 + x + 3$

where y and x are measured in feet. Will the baseball clear a 10-foot fence located 300 feet from home plate?

Algebraic Solution

The height of the baseball is a function of the horizontal distance from home plate. When $x = 300$, you can find the height of the baseball as follows.

$f(x) = -0.0032x^2 + x + 3$ Write original function.

$f(300) = -0.0032(300)^2 + 300 + 3$ Substitute 300 for x.

$= 15$ Simplify.

When $x = 300$, the height of the baseball is 15 feet, so the baseball will clear a 10-foot fence.

Graphical Solution

Use a graphing utility to graph the function $y = -0.0032x^2 + x + 3$. Use the *value* feature or the *zoom* and *trace* features of the graphing utility to estimate that $y = 15$ when $x = 300$, as shown in Figure 1.18. So, the ball will clear a 10-foot fence.

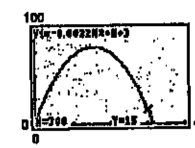


Figure 1.18

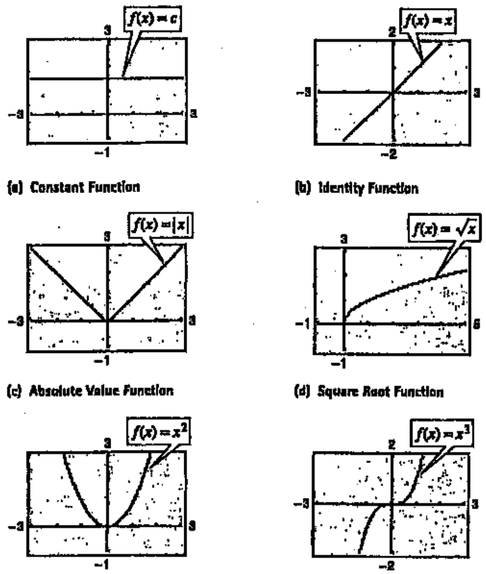
✓ *Checkpoint* Now try Exercise 81.

1.4 Shifting, Reflecting, and Stretching Graphs

Summary of Graphs of Common Functions

One of the goals of this text is to enable you to build your intuition for the basic shapes of the graphs of different types of functions. For instance, from your study of lines in Section 1.1, you can determine the basic shape of the graph of the linear function $f(x) = mx + b$. Specifically, you know that the graph of this function is a line whose slope is m and whose y-intercept is $(0, b)$.

The six graphs shown in Figure 1.40 represent the most commonly used functions in algebra. Familiarity with the basic characteristics of these simple graphs will help you analyze the shapes of more complicated graphs.



(a) Constant Function (b) Identity Function (c) Absolute Value Function (d) Square Root Function (e) Quadratic Function (f) Cubic Function

Figure 1.40

Throughout this section, you will discover how many complicated graphs are derived by shifting, stretching, shrinking, or reflecting the common graphs shown above. Shifts, stretches, shrinks, and reflections are called *transformations*. Many graphs of functions can be created from a combination of these transformations.

What you should learn

- Recognize graphs of common functions.
- Use vertical and horizontal shifts and reflections to graph functions.
- Use nonrigid transformations to graph functions.

Why you should learn it

Recognizing the graphs of common functions and knowing how to shift, reflect, and stretch graphs of functions can help you sketch a wide variety of simple functions by hand. This skill is useful in sketching graphs of functions that model real-life data. For example, in Exercise 67 on page 50, you are asked to sketch a function that models the amount of fuel used by trucks from 1980 through 2000.

Index Stock

Vertical and Horizontal Shifts

Many functions have graphs that are simple transformations of the common graphs summarized in Figure 1.40. For example, you can obtain the graph of

$$h(x) = x^2 + 2$$

by shifting the graph of $f(x) = x^2$ *upward* two units, as shown in Figure 1.41. In function notation, h and f are related as follows.

$$h(x) = x^2 + 2$$
$$= f(x) + 2 \qquad \text{Upward shift of two units}$$

Similarly, you can obtain the graph of

$$g(x) = (x - 2)^2$$

by shifting the graph of $f(x) = x^2$ to the *right* two units, as shown in Figure 1.42. In this case, the functions g and f have the following relationship.

$$g(x) = (x - 2)^2$$
$$= f(x - 2) \qquad \text{Right shift of two units}$$

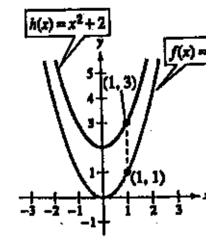


Figure 1.41 Vertical shift upward: two units

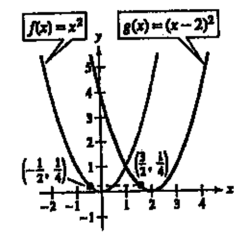


Figure 1.42 Horizontal shift to the right: two units

The following list summarizes horizontal and vertical shifts.

> **Vertical and Horizontal Shifts**
>
> Let c be a positive real number. Vertical and horizontal shifts in the graph of $y = f(x)$ are represented as follows.
>
> 1. Vertical shift c units *upward*: $h(x) = f(x) + c$
> 2. Vertical shift c units *downward*: $h(x) = f(x) - c$
> 3. Horizontal shift c units to the *right*: $h(x) = f(x - c)$
> 4. Horizontal shift c units to the *left*: $h(x) = f(x + c)$

In items 3 and 4, be sure you see that $h(x) = f(x - c)$ corresponds to a *right* shift and $h(x) = f(x + c)$ corresponds to a *left* shift for $c > 0$.

> **Exploration**
>
> Use a graphing utility to display (in the same viewing window) the graphs of $y = x^2 + c$, where $c = -2, 0, 2$, and 4. Use the result to describe the effect that c has on the graph.
>
> Use a graphing utility to display (in the same viewing window) the graphs of $y = (x + c)^2$, where $c = -2, 0, 2$, and 4. Use the result to describe the effect that c has on the graph.

Example 1 Shifts in the Graph of a Function

Compare the graph of each function with the graph of $f(x) = x^3$.

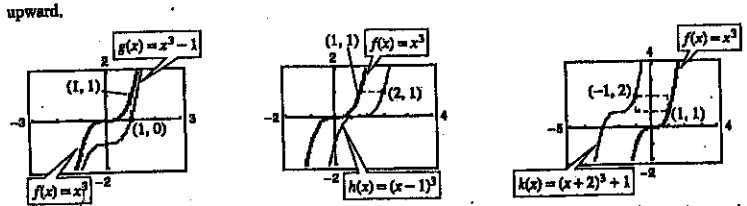
a. $g(x) = x^3 - 1$ b. $h(x) = (x - 1)^3$ c. $k(x) = (x + 2)^3 + 1$

Solution

a. Graph $f(x) = x^3$ and $g(x) = x^3 - 1$ [see Figure 1.43(a)]. You can obtain the graph of g by shifting the graph of f one unit downward.

b. Graph $f(x) = x^3$ and $h(x) = (x - 1)^3$ [see Figure 1.43(b)]. You can obtain the graph of h by shifting the graph of f one unit to the right.

c. Graph $f(x) = x^3$ and $k(x) = (x + 2)^3 + 1$ [see Figure 1.43(c)]. You can obtain the graph of k by shifting the graph of f two units to the left and then one unit upward.



(a) Vertical shift: one unit downward (b) Horizontal shift: one unit right (c) Two units left and one unit upward

Figure 1.43

Checkpoint Now try Exercise 3.

Example 2 Finding Equations from Graphs

The graph of $f(x) = x^2$ is shown in Figure 1.44. Each of the graphs in Figure 1.45 is a transformation of the graph of f. Find an equation for each function.

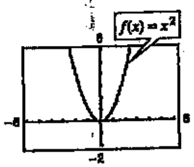
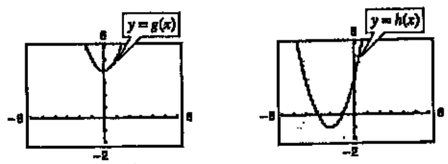


Figure 1.44



(a) (b)

Figure 1.45

Solution

a. The graph of g is a vertical shift of four units upward of the graph of $f(x) = x^2$. So, the equation for g is $g(x) = x^2 + 4$.

b. The graph of h is a horizontal shift of two units to the left, and a vertical shift of one unit downward, of the graph of $f(x) = x^2$. So, the equation for h is $h(x) = (x + 2)^2 - 1$.

Checkpoint Now try Exercise 21.

Reflecting Graphs

The second common type of transformation is called a reflection. For instance, if you consider the x-axis to be a mirror, the graph of $h(x) = -x^2$ is the mirror image (or reflection) of the graph of $f(x) = x^2$ (see Figure 1.46).

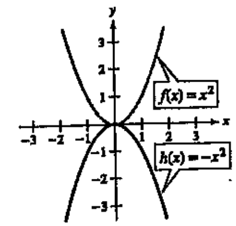


Figure 1.46

Exploration

Compare the graph of each function with the graph of $f(x) = x^2$ by using a graphing utility to graph the function and f in the same viewing window. Describe the transformation.

a. $g(x) = -x^2$

b. $h(x) = (-x)^2$

Reflections in the Coordinate Axes

Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$
2. Reflection in the y-axis: $h(x) = f(-x)$

Example 3 Finding Equations from Graphs

The graph of $f(x) = x^4$ is shown in Figure 1.47. Each of the graphs in Figure 1.48 is a transformation of the graph of f. Find an equation for each function.

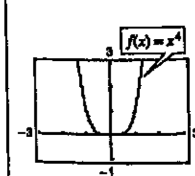
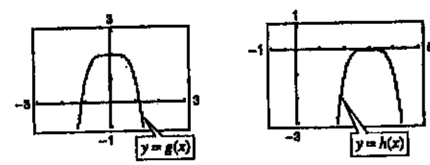


Figure 1.47



(a) (b)

Figure 1.48

Solution

a. The graph of g is a reflection in the x-axis *followed by* an upward shift of two units of the graph of $f(x) = x^4$. So, the equation for g is $g(x) = -x^4 + 2$.

b. The graph of h is a horizontal shift of three units to the right *followed by* a reflection in the x-axis of the graph of $f(x) = x^4$. So, the equation for h is $h(x) = -(x - 3)^4$.

Checkpoint Now try Exercise 25.

Example 4 Reflections and Shifts

Compare the graph of each function with the graph of $f(x) = \sqrt{x}$.

a. $g(x) = -\sqrt{x}$ **b.** $h(x) = \sqrt{-x}$ **c.** $k(x) = -\sqrt{x+2}$

Algebraic Solution

a. Relative to the graph of $f(x) = \sqrt{x}$, the graph of g is a reflection in the x-axis because

$$g(x) = -\sqrt{x}$$
$$= -f(x).$$

b. The graph of h is a reflection of the graph of $f(x) = \sqrt{x}$ in the y-axis because

$$h(x) = \sqrt{-x}$$
$$= f(-x).$$

c. From the equation

$$k(x) = -\sqrt{x+2}$$
$$= -f(x+2)$$

you can conclude that the graph of k is a left shift of two units, followed by a reflection in the x-axis, of the graph of $f(x) = \sqrt{x}$.

Graphical Solution

a. Use a graphing utility to graph f and g in the same viewing window. From the graph in Figure 1.49, you can see that the graph of g is a reflection of the graph of f in the x-axis.

b. Use a graphing utility to graph f and h in the same viewing window. From the graph in Figure 1.50, you can see that the graph of h is a reflection of the graph of f in the y-axis.

c. Use a graphing utility to graph f and k in the same viewing window. From the graph in Figure 1.51, you can see that the graph of k is a left shift of two units of the graph of f, followed by a reflection in the x-axis.

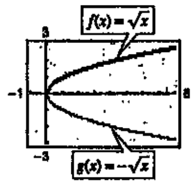


Figure 1.49

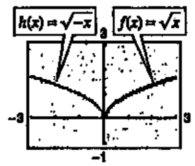


Figure 1.50

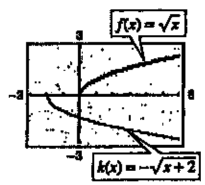


Figure 1.51

✓ *Checkpoint* Now try Exercise 27.

When graphing functions involving square roots, remember that the domain must be restricted to exclude negative numbers inside the radical. For instance, here are the domains of the functions in Example 4.

Domain of $g(x) = -\sqrt{x}$: $x \ge 0$

Domain of $h(x) = \sqrt{-x}$: $x \le 0$

Domain of $k(x) = -\sqrt{x+2}$: $x \ge -2$

Nonrigid Transformations

Horizontal shifts, vertical shifts, and reflections are called **rigid transformations** because the basic shape of the graph is unchanged. These transformations change only the *position* of the graph in the coordinate plane. **Nonrigid transformations** are those that cause a *distortion*—a change in the shape of the original graph. For instance, a nonrigid transformation of the graph of $y = f(x)$ is represented by $y = cf(x)$ (each y-value is multiplied by c), where the transformation is a **vertical stretch** if $c > 1$ and a **vertical shrink** if $0 < c < 1$. Another nonrigid transformation of the graph of $y = f(x)$ is represented by $h(x) = f(cx)$ (each x-value is multiplied by $1/c$), where the transformation is a **horizontal shrink** if $c > 1$ and a **horizontal stretch** if $0 < c < 1$.

Example 5 Nonrigid Transformations

Compare the graph of each function with the graph of $f(x) = |x|$.

a. $h(x) = 3|x|$ **b.** $g(x) = \frac{1}{3}|x|$

Solution

a. Relative to the graph of $f(x) = |x|$, the graph of

$$h(x) = 3|x|$$
$$= 3f(x)$$

is a vertical stretch (each y-value is multiplied by 3) of the graph of f. (See Figure 1.52.)

b. Similarly, the graph of

$$g(x) = \frac{1}{3}|x|$$
$$= \frac{1}{3}f(x)$$

is a vertical shrink (each y-value is multiplied by $\frac{1}{3}$) of the graph of f. (See Figure 1.53.)

✓ *Checkpoint* Now try Exercise 37.

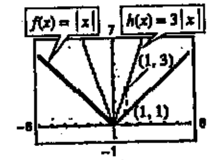


Figure 1.52

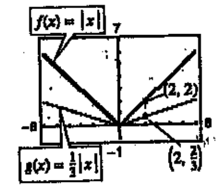


Figure 1.53

Example 6 Nonrigid Transformations

Compare the graph of $h(x) = f(\frac{1}{2}x)$ with the graph of $f(x) = 2 - x^3$.

Solution

Relative to the graph of $f(x) = 2 - x^3$, the graph of

$$h(x) = f(\tfrac{1}{2}x) = 2 - (\tfrac{1}{2}x)^3 = 2 - \tfrac{1}{8}x^3$$

is a horizontal stretch (each x-value is multiplied by 2) of the graph of f. (See Figure 1.54.)

✓ *Checkpoint* Now try Exercise 43.

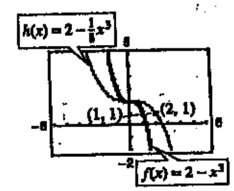


Figure 1.54

1 Review Exercises

1.1 In Exercises 1–6, [redacted] find the slope of the line passing through the pair of points.

1. $(-3, 2), (8, 2)$
2. $(7, -1), (7, 12)$
3. $\left(\frac{1}{2}, 1\right), \left(5, \frac{5}{2}\right)$
4. $\left(-\frac{3}{4}, \frac{5}{6}\right), \left(\frac{1}{2}, -\frac{5}{2}\right)$
5. $(-4.5, 6), (2.1, 3)$
6. $(-2.7, -6.3), (-1, -1.2)$

In Exercises 7–16, use the point on the line and the slope of the line to find the general form of the equation of the line, [redacted]

	Point	Slope
7.	$(2, -1)$	$m = \frac{1}{4}$
8.	$(-3, 5)$	$m = -\frac{1}{2}$
9.	$(0, -5)$	$m = \frac{3}{2}$
10.	$(3, 0)$	$m = -\frac{2}{3}$
11.	$\left(\frac{1}{5}, -5\right)$	$m = -1$
12.	$\left(0, \frac{7}{8}\right)$	$m = -\frac{4}{5}$
13.	$(-2, 6)$	$m = 0$
14.	$(-8, 8)$	$m = 0$
15.	$(10, -6)$	m is undefined.
16.	$(5, 4)$	m is undefined.

In Exercises 17–20, find the slope-intercept form of the equation of the line that passes through the points. [redacted]

17. $(2, -1), (4, -1)$
18. $(0, 0), (0, 10)$
19. $(-1, 0), (6, 2)$
20. $(1, 6), (4, 2)$

Rate of Change In Exercises 21 and 22, you are given the dollar value of a product in 2005 *and* the rate at which the value of the item is expected to change during the 5 years following. Use this information to write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 5$ represent 2005.)

	2005 Value	Rate
21.	\$12,500	\$850 increase per year
22.	\$72.95	\$5.15 decrease per year

23. ***Sales*** During the second and third quarters of the year, an e-commerce business had sales of \$160,000 and \$185,000, respectively. The growth of sales follows a linear pattern. Estimate sales during the fourth quarter.

24. ***Depreciation*** The dollar value of a VCR in 2004 is \$85, and the product will decrease in value at an expected rate of \$10.75 per year.
   (a) Write a linear equation that gives the dollar value V of the VCR in terms of the year t. (Let $t = 4$ represent 2004.)
   (b) Use a graphing utility to graph the equation found in part (a).
   (c) Use the *value* or *trace* feature of your graphing utility to estimate the dollar value of the VCR in 2008.

In Exercises 25–28, write the slope-intercept forms of the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line. Verify your result with a graphing utility (use a square setting).

	Point	Line
25.	$(3, -2)$	$5x - 4y = 8$
26.	$(-8, 3)$	$2x + 3y = 5$
27.	$(-6, 2)$	$x = 4$
28.	$(3, -4)$	$y = 2$

1.2 In Exercises 29 and 30, which sets of ordered pairs represent functions from A to B? Explain.

29. $A = \{10, 20, 30, 40\}$ and $B = \{0, 2, 4, 6\}$
   (a) $\{(20, 4), (40, 0), (20, 6), (30, 2)\}$
   (b) $\{(10, 4), (20, 4), (30, 4), (40, 4)\}$
   (c) $\{(40, 0), (30, 2), (20, 4), (10, 6)\}$
   (d) $\{(20, 2), (10, 0), (40, 4)\}$

30. $A = \{u, v, w\}$ and $B = \{-2, -1, 0, 1, 2\}$
   (a) $\{(v, -1), (u, 2), (w, 0), (u, -2)\}$
   (b) $\{(u, -2), (v, 2), (w, 1)\}$
   (c) $\{(u, 2), (v, 2), (w, 1), (w, 1)\}$
   (d) $\{(w, -2), (v, 0), (w, 2)\}$

In Exercises 31–34, determine whether the equation represents y as a function of x.

31. $16x - y^4 = 0$
32. $2x - y - 3 = 0$
33. $y = \sqrt{1 - x}$
34. $|y| = x + 2$

In Exercises 35–38, evaluate the function at each value of the independent variable and simplify.

35. $f(x) = x^2 + 1$
   (a) $f(2)$ (b) $f(-4)$
   (c) $f(t^2)$ (d) $-f(x)$

36. $g(x) = x^{4/3}$
   (a) $g(8)$ (b) $g(t + 1)$
   (c) $g(-27)$ (d) $g(-x)$

37. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$
   (a) $h(-2)$ (b) $h(-1)$
   (c) $h(0)$ (d) $h(2)$

38. $f(x) = \dfrac{3}{2x - 5}$
   (a) $f(1)$ (b) $f(-2)$
   (c) $f(t)$ (d) $f(10)$

In Exercises 39–44, find the domain of the function.

39. $f(x) = (x - 1)(x + 2)$
40. $f(x) = x^2 - 4x - 32$
41. $f(x) = \sqrt{25 - x^2}$
42. $f(x) = \sqrt{x^2 + 8x}$
43. $g(s) = \dfrac{5}{3s - 9}$
44. $f(x) = \dfrac{2}{3x + 4}$

45. ***Cost*** A hand tool manufacturer produces a product for which the variable cost is \$5.35 per unit and the fixed costs are \$16,000. The company sells the product for \$8.20 and can sell all that it produces.
   (a) Write the total cost C as a function of x, the number of units produced.
   (b) Write the profit P as a function of x.

46. ***Consumerism*** The retail sales R (in billions of dollars) of lawn care products and services in the United States from 1994 to 2001 can be approximated by the model

$$R(t) = \begin{cases} -0.67t + 11.0, & 4 \le t \le 7 \\ 0.600t^2 - 10.06t + 50.7, & 8 \le t \le 11 \end{cases}$$

where t represents the year, with $t = 4$ corresponding to 1994. Use the *table* feature of a graphing utility to approximate the retail sales of lawn care products and services for each year from 1994 to 2001. (Source: The National Gardening Association)

In Exercises 47 and 48, find the difference quotient and simplify your answer.

47. $f(x) = 2x^2 + 3x - 1, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \ne 0$
48. $f(x) = x^3 - 5x^2 + x, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \ne 0$

1.3 In Exercises 49–52, use a graphing utility to graph the function and estimate its domain and range. Then find the domain and range algebraically.

49. $f(x) = 3 - 2x^2$
50. $f(x) = \sqrt{2x^2 - 1}$
51. $h(x) = \sqrt{36 - x^2}$
52. $g(x) = |x + 5|$

In Exercises 53–56, (a) use a graphing utility to graph the equation and (b) use the Vertical Line Test to determine whether y is a function of x.

53. $y = \dfrac{x^2 + 3x}{6}$
54. $y = -\dfrac{2}{3}|x + 5|$
55. $3x + y^2 = 2$
56. $x^2 + y^2 = 49$

In Exercises 57–60, (a) use a graphing utility to graph the function and (b) determine the open intervals on which the function is increasing, decreasing, or constant.

57. $f(x) = x^3 - 3x$
58. $f(x) = \sqrt{x^2 - 9}$
59. $f(x) = x\sqrt{x - 6}$
60. $f(x) = \dfrac{|x + 8|}{2}$

In Exercises 61–64, use a graphing utility to approximate (to two decimal places) any relative minimum or maximum values of the function.

61. $f(x) = (x^2 - 4)^2$
62. $f(x) = x^2 - x - 1$
63. $h(x) = 4x^3 - x^4$
64. $f(x) = x^3 - 4x^2 - 1$

In Exercises 65 and 66, sketch the graph of the piecewise-defined function by hand.

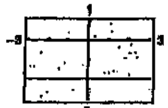
65. $f(x) = \begin{cases} 3x + 5, & x < 0 \\ x - 4, & x \ge 0 \end{cases}$
66. $f(x) = \begin{cases} x^2 + 7, & x < 1 \\ x^2 - 5x + 6, & x \ge 1 \end{cases}$

In Exercises 67 and 68, algebraically determine whether the function is even, odd, or neither. Verify your answer using a graphing utility.

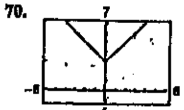
67. $f(x) = (x^2 - 8)^2$ 68. $f(x) = 2x^3 - x^2$

1.4 In Exercises 69–72, identify the original [redacted] function and describe the transformation shown in the graph. Write an equation for the graphed function.

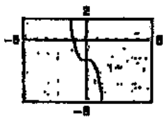
69.



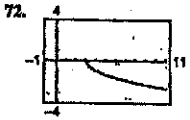
70.



71.



72.



In Exercises 73–84, h is related to one of the six common functions on page 42. (a) Identify the common function f. (b) Describe the sequence of transformations from f to h. (c) Sketch the graph of h by hand. (d) Use function notation to write h in terms of the common function f.

73. $h(x) = x^2 - 6$ 74. $h(x) = (x - 3)^2 - 2$

75. $h(x) = (x - 1)^3 + 7$ 76. $h(x) = (x + 2)^3 + 5$

77. $h(x) = \sqrt{x} - 5$ 78. $h(x) = |x + 8| - 1$

79. $h(x) = -x^2 - 3$ 80. $h(x) = -(x - 2)^2 - 8$

81. $h(x) = -2x^2 + 3$ 82. $h(x) = \frac{1}{2}(x - 3)^2 + 6$

83. $h(x) = -\frac{1}{2}|x| + 9$ 84. $h(x) = \sqrt{3x} - 5$

1.5 In Exercises 85–94, let $f(x) = 3 - 2x$, $g(x) = \sqrt{x}$, and $h(x) = 3x^2 + 2$, and find the indicated values.

85. $(f - g)(4)$ 86. $(f + h)(5)$

87. $(f + g)(25)$ 88. $(g - h)(1)$

89. $(fh)(1)$ 90. $\left(\frac{g}{h}\right)(1)$

91. $(h \circ g)(7)$ 92. $(g \circ f)(-2)$

93. $(f \circ h)(-4)$ 94. $(g \circ h)(6)$

Data Analysis In Exercises 95 and 96, the numbers (in thousands) of students taking the SAT (y_1) and ACT (y_2) for the years 1996 through 2001 can be modeled by $y_1 = -2.75t^2 + 86.8t + 659$ and $y_2 = -1.88t^2 + 62.4t + 616$, where t represents the year, with $t = 6$ corresponding to 1996. (Source: College Entrance Examination Board and ACT, Inc.)

95. Use a graphing utility to graph y_1, y_2, and $y_1 + y_2$ in the same viewing window.

96. Use the model $y_1 + y_2$ to estimate the total number of students taking the SAT and ACT in 2006.

1.6 In Exercises 97 and 98, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

97. $f(x) = 6x$ 98. $f(x) = x + 5$

In Exercises 99 and 100, show that f and g are inverse functions (a) graphically and (b) numerically.

99. $f(x) = 3 - 4x$, $g(x) = \dfrac{3 - x}{4}$

100. $f(x) = \sqrt{x + 1}$, $g(x) = x^2 - 1, x \geq 0$

In Exercises 101–104, use a graphing utility to graph the function and use the Horizontal Line Test to determine whether the function is one-to-one and so has an inverse function.

101. $f(x) = \frac{1}{2}x - 3$ 102. $f(x) = (x - 1)^2$

103. $h(t) = \dfrac{2}{t - 3}$ 104. $g(x) = \sqrt{x + 6}$

In Exercises 105–108, find the inverse function of f algebraically.

105. $f(x) = \dfrac{x}{12}$ 106. $f(x) = \dfrac{7x + 3}{8}$

107. $f(x) = 4x^3 - 3$ 108. $f(x) = \sqrt{x + 10}$

1.7

109. *Education* The following ordered pairs give the entrance exam scores x and the grade-point averages y after 1 year of college for 10 students.

(75, 2.3), (82, 3.0), (90, 3.6), (65, 2.0), (70, 2.1), (88, 3.5), (93, 3.9), (69, 2.0), (80, 2.8), (85, 3.3)

(a) Create a scatter plot for the data.

(b) Does the relationship between x and y appear to be approximately linear? Explain.

110. *Stress Test* A machine part was tested by bending it x centimeters 10 times per minute until it failed (y equals the time to failure in hours). The results are given as the following ordered pairs.

(3, 61), (6, 56), (9, 53), (12, 55), (15, 48), (18, 35), (21, 36), (24, 33), (27, 44), (30, 23)

(a) Create a scatter plot for the data.

(b) Does the relationship between x and y appear to be approximately linear? If not, give some possible explanations.

111. *Falling Object* In an experiment, students measured the speed s (in meters per second) of a ball t seconds after it was released. The results are shown in the table.

Time, t	Speed, s
0	0
1	11.0
2	19.4
3	29.2
4	39.4

(a) Sketch a scatter plot of the data.

(b) Find the equation of the line that seems to best fit the data.

(c) Use the *regression* feature of a graphing utility to find a linear model for the data. Compare with the model in part (b).

(d) Use the model in part (c) to estimate the speed of the ball after 2.5 seconds.

112. *Sales* The table shows the sales S (in millions of dollars) for Timberland from 1995 to 2002. (Source: The Timberland Co.)

Year	Sales, S
1995	655.1
1996	690.0
1997	796.5
1998	862.2
1999	917.2
2000	1091.5
2001	1183.6
2002	1190.9

(a) Use the *regression* feature of a graphing utility to find a linear model for the data. Let t represent the year, with $t = 5$ corresponding to 1995.

(b) Use a graphing utility to plot the data and graph the model in the same viewing window.

(c) Interpret the slope of the model in the context of the problem.

(d) Use the model to find the year in which the sales will exceed $1300 million.

(e) Create a table showing the actual values of S and the values of S given by the model. How closely does the model represent the data?

Height In Exercises 113–116, the following ordered pairs (x, y) represent the percent y of women between the ages of 20 and 29 who are under a certain height x (in feet). (Source: U.S. National Center for Health Statistics)

(4.67, 0.6) (5.17, 21.8) (5.67, 92.4)
(4.75, 0.7) (5.25, 34.3) (5.75, 96.2)
(4.83, 1.2) (5.33, 48.9) (5.83, 98.6)
(4.92, 3.1) (5.42, 62.7) (5.92, 99.5)
(5.00, 6.0) (5.50, 74.0) (6.00, 100.0)
(5.08, 11.5) (5.58, 84.7)

113. Use the *regression* feature of a graphing utility to find a linear model for the data.

114. Use a graphing utility to plot the data and graph the model in the same viewing window.

115. How closely does the model fit the data?

116. Can the model be used to estimate the percent of women who are under a height of greater than 6 feet?

Synthesis

True or False? In Exercises 117–120, determine whether the statement is true or false. Justify your answer.

117. If the graph of the common function $f(x) = x^2$ is moved six units to the right, moved three units upward, and reflected in the x-axis, then the point $(-1, 28)$ will lie on the graph of the transformation.

118. If $f(x) = x^n$ where n is odd, f^{-1} exists.

119. There exists no function f such that $f = f^{-1}$.

120. The sign of the slope of a regression line is always positive.